CS141: Introduction to Artificical Intelligence

Homework 8

Andrew Chin *Due: April 11, 2007*

8.1

a)

$$\forall x, y : Meet(Dan, Joe) \wedge$$
$$Meet(Spiros, Joe) \wedge$$
$$(\neg Meet(x, Joe) \vee MustEatMushrooms(x)) \wedge$$
$$(\neg MustEatMushrooms(x) \vee HatesJoe(x)) \wedge$$
$$(\neg HatesJoe(x) \vee \neg HatesJoe(y) \vee LikesEachOther(x, y))$$

b)
1. $Meet(Dan, Joe)$ (And-Elimination from KB)
2. $Meet(Spiros, Joe)$ (And-Elimination from KB)
3. $\neg Meet(x, Joe) \vee MustEatMushrooms(x)$ (And-Elimination from KB)
4. $\neg Meet(Dan, Joe) \vee MustEatMushrooms(Dan)$ (Universal Instantiation)
5. $MustEatsMushrooms(Dan)$ (Disjunctive syllogism)
6. $\neg MustEatMushrooms(x) \vee HatesJoe(x))$ (And-Elimination from KB)
7. $\neg MustEatMushrooms(Dan) \vee HatesJoe(Dan)$ (Universal Instantiation)
8. $HatesJoe(Dan)$ (Disjunctive syllogism)
9. $\neg Meet(Spiros, Joe) \vee MustEatMushrooms(Spiros)$ (Universal Instantiation)
10. $MustEatsMushrooms(Spiros)$ (Disjunctive syllogism)
11. $\neg MustEatMushrooms(Spiros) \vee HatesJoe(Spiros)$ (Universal Instantiation)
12. $HatesJoe(Spiros)$ (Disjunctive syllogism)
13. $\neg HatesJoe(x) \vee \neg HatesJoe(y) \vee LikesEachOther(x, y)$ (And-Elimination)
14. $\neg HatesJoe(Dan) \vee \neg HatesJoe(Spiros) \vee LikesEachOther(Dan, Spiros)$ (Universal Instantiation)
15. $\neg HatesJoe(Spiros) \vee LikesEachOther(Dan, Spiros)$ (Disjunctive syllogism)
16. $LikesEachOther(Dan, Spiros)$ (Disjunctive syllogism)

8.3

Note: lists are shows in square brackets for readability

a)

$$
\begin{aligned}
&\texttt{UNIFY}(p(f(f(f(a,x_0),x_1),x_2)),p(f(x_3,f(x_4,f(x_5,a)))))\\
=&\texttt{UNIFY}([f(f(f(a,x_0),x_1),x_2)],[f(x_3,f(x_4,f(x_5,a)))],\texttt{UNIFY}(p,p))\\
=&\texttt{UNIFY}([f(f(f(a,x_0),x_1),x_2)],[f(x_3,f(x_4,f(x_5,a)))])\\
=&\texttt{UNIFY}([],[],\texttt{UNIFY}(f(f(f(a,x_0),x_1),x_2),f(x_3,f(x_4,f(x_5,a)))))\\
=&\texttt{UNIFY}(f(f(f(a,x_0),x_1),x_2),f(x_3,f(x_4,f(x_5,a))))\\
=&\texttt{UNIFY}([f(f(a,x_0),x_1),x_2],[x_3,f(x_4,f(x_5,a))],\texttt{UNIFY}(f,f))\\
=&\texttt{UNIFY}([f(f(a,x_0),x_1),x_2],[x_3,f(x_4,f(x_5,a))])\\
=&\texttt{UNIFY}(x_2,f(x_4,f(x_5,a)),\texttt{UNIFY}(f(f(a,x_0),x_1),x_3))\\
&\texttt{UNIFY}(f(f(a,x_0),x_1),x_3)=\texttt{UNIFYVAR}(x_3,f(f(a,x_0),x_1))\\
&=\{x_3/f(f(a,x_0),x_1)\}\\
=&\texttt{UNIFY}(x_2,f(x_4,f(x_5,a)),\{x_3/f(f(a,x_0),x_1)\})\\
=&\texttt{UNIFYVAR}(x_2,f(x_4,f(x_5,a)),\{x_3/f(f(a,x_0),x_1)\})\\
=&\{x_3/f(f(a,x_0),x_1),x_2/f(x_4,f(x_5,a))\}
\end{aligned}
$$

b)

$$
\begin{aligned}
&\texttt{UNIFY}(p(x),p(f(y)))\\
=&\texttt{UNIFY}(x,f(y),\texttt{UNIFY}(p,p))\\
=&\texttt{UNIFY}(x,f(y))\\
=&\texttt{UNIFYVAR}(x,f(y))\\
=&x/f(y)
\end{aligned}
$$

c)

$$
\begin{aligned}
&\texttt{UNIFY}(p(x,h(x,y),y),p(x',z,b))\\
=&\texttt{UNIFY}([x,h(x,y),y],[x',z,b],\texttt{UNIFY}(p,p))\\
=&\texttt{UNIFY}([x,h(x,y),y],[x',z,b])\\
=&\texttt{UNIFY}([h(x,y),y],[z,b],\texttt{UNIFY}(x,x'))\\
&\texttt{UNIFY}(x,x')=\texttt{UNIFYVAR}(x,x')=\{x/x'\}\\
=&\texttt{UNIFY}([h(x,y),y],[z,b],\{x/x'\})\\
=&\texttt{UNIFY}(y,b,\texttt{UNIFY}(h(x,y),z,\{x/x'\}))\\
&\texttt{UNIFY}(h(x,y),z,\{x/x'\})=\texttt{UNIFYVAR}(z,h(x,y),\{x/x'\})=\\
&\{x/x',z/h(x,y)\}\\
=&\texttt{UNIFY}(y,b,\texttt{UNIFY}(h(x,y),z,\{x/x',z/h(x,y)\}))\\
=&\{x/x',z/h(x,y),y/b\}
\end{aligned}
$$

d)

$$
\begin{aligned}
&\texttt{UNIFY}(p(x,x,y,y), p(x', f(y'), y', f(x'))) \\
=&\texttt{UNIFY}([x,x,y,y],[x', f(y'), y', f(x')], \texttt{UNIFY}(p,p))) \\
&\texttt{UNIFY}(p,p) = empty \\
=&\texttt{UNIFY}([x,x,y,y],[x', f(y'), y', f(x')]) \\
=&\texttt{UNIFY}([x,y,y],[f(y'), y', f(x')], \texttt{UNIFY}(x,x')) \\
&\texttt{UNIFY}(x,x') = \texttt{UNIFYVAR}(x,x') = \{x/x'\} \\
=&\texttt{UNIFY}([x,y,y],[f(y'), y', f(x')], \{x/x'\}) \\
=&\texttt{UNIFY}([y,y],[y', f(x')], \texttt{UNIFY}(x, f(y'), \{x/x'\})) \\
&\texttt{UNIFY}(x, f(y'), \{x/x'\}) = \texttt{UNIFYVAR}(x, f(y'), \{x/x'\}) = \texttt{UNIFY}(x', f(y'), \{x/x'\}) \\
&= \texttt{UNIFYVAR}(x', f(y'), \{x/x'\}) = \{x/x', x'/f(y')\} \\
=&\texttt{UNIFY}([y,y],[y', f(x')], \{x/x', x'/f(y')\}) \\
=&\texttt{UNIFY}(y, f(x'), \texttt{UNIFY}(y, y', \{x/x', x'/f(y')\})) \\
=&\texttt{UNIFY}(y, f(x'), \texttt{UNIFYVAR}(y, y', \{x/x', x'/f(y')\})) \\
=&\texttt{UNIFY}(y, f(x'), \{x/x', x'/f(y')\}) \\
=&\texttt{UNIFY}(y, f(x'), \{x/x', x'/f(y')\}) \\
=&\texttt{UNIFYVAR}(y, f(x'), \{x/x', x'/f(y')\}) \\
=&\texttt{UNIFY}(y', f(x'), \{x/x', x'/f(y')\}) \\
=&\texttt{UNIFYVAR}(y', f(x'), \{x/x', x'/f(y')\}) \\
=&\{x/x', x'/f(y'), y'/f(x')\}
\end{aligned}
$$

8.4

k=0

$\pi(s) = \beta$
$U(s) = 0$
Updated $U(s)$:
$U(1A) = \sum_{s'}(R(1A,\beta,s') + U(s'))T(1A,\beta,s')$
$U(1A) = (R(1A,\beta,T) + U(T))T(1A,\beta,T) = (100+0)\times 1 = 100$
$U(1B) = (R(1B,\beta,T) + U(T))T(1B,\beta,T) = (200+0)\times 1 = 200$
$U(2A) = (R(2A,\beta,T) + U(T))T(2A,\beta,T) = (50+0)\times 1 = 50$
$U(2B) = (R(2B,\beta,T) + U(T))T(2B,\beta,T) = (400+0)\times 1 = 400$
$U(3A) = (R(3A,\beta,T) + U(T))T(3A,\beta,T) = (0+0)\times 1 = 0$
$U(3B) = (R(3B,\beta,T) + U(T))T(3B,\beta,T) = (600+0)\times 1 = 600$
$U(T) = (R(T,\beta,T) + U(T))T(T,\beta,T) = (0+0)\times 1 = 0$

Update policy $\pi(s)$ for all states s:

$$
\begin{aligned}
\pi(1A) &= argmax_a \textstyle\sum_{s'}(R(1A,a,s') + U(s')) \times T(1A,a,s') \\
a = \alpha &: = (R(1A,\alpha,2A) + U(2A)) \times T(1A,\alpha,2A) + (R(1A,\alpha,2B) + U(2B)) \times T(1A,\alpha,2B) \\
&= (0+50)\times 0.4 + (0+400)\times 0.6 = 260 \\
a = \beta &: = (R(1A,\beta,T) + U(T)) \times T(1A,\beta,T) \\
&= (100+0)\times 1 = 100 \\
\pi(1A) &:= \alpha \\
\pi(1B) &= argmax_a \textstyle\sum_{s'}(R(1B,a,s') + U(s')) \times T(1B,a,s')
\end{aligned}
$$

$$a = \alpha \ : = (R(1B, \alpha, 2A) + U(2A)) \times T(1B, \alpha, 2A) + (R(1B, \alpha, 2B) + U(2B)) \times T(1B, \alpha, 2B)$$
$$= (0 + 50) \times 0.6 + (0 + 400) \times 0.4 = 190$$
$$a = \beta \ : = (R(1B, \beta, T) + U(T)) \times T(1B, \beta, T)$$
$$= (200 + 0) \times 1 = 200$$
$$\pi(1B) := \beta$$

$$\pi(2A) = argmax_a \sum_{s'} (R(2A, a, s') + U(s')) \times T(2A, a, s')$$
$$a = \alpha \ : = (R(2A, \alpha, 3A) + U(3A)) \times T(2A, \alpha, 3A) + (R(2A, \alpha, 3B) + U(3B)) \times T(2A, \alpha, 3B)$$
$$= (0 + 0) \times 0.4 + (0 + 600) \times 0.6 = 360$$
$$a = \beta \ : = (R(2A, \beta, T) + U(T)) \times T(2A, \beta, T)$$
$$= (50 + 0) \times 1 = 50$$
$$\pi(2A) := \alpha$$

$$\pi(2B) = argmax_a \sum_{s'} (R(2B, a, s') + U(s')) \times T(2B, a, s')$$
$$a = \alpha \ : = (R(2B, \alpha, 3A) + U(3A)) \times T(2B, \alpha, 3A) + (R(2B, \alpha, 3B) + U(3B)) \times T(2B, \alpha, 3B)$$
$$= (0 + 0) \times 0.6 + (0 + 600) \times 0.4 = 240$$
$$a = \beta \ : = (R(2B, \beta, T) + U(T)) \times T(2B, \beta, T)$$
$$= (400 + 0) \times 1 = 400$$
$$\pi(2B) := \beta$$

$$\pi(3A) = argmax_a \sum_{s'} (R(3A, a, s') + U(s')) \times T(3A, a, s')$$
$$a = \alpha \ : = (R(3A, \alpha, T) + U(T)) \times T(3A, \alpha, T)$$
$$= (0 + 0) \times 1 = 0$$
$$a = \beta \ : = (R(3A, \beta, T) + U(T)) \times T(3A, \beta, T)$$
$$= (0 + 0) \times 1 = 0$$
$$\pi(3A) := \beta$$

$$\pi(3B) = argmax_a \sum_{s'} (R(3B, a, s') + U(s')) \times T(3B, a, s')$$
$$a = \alpha \ : = (R(3B, \alpha, T) + U(T)) \times T(3B, \alpha, T)$$
$$= (0 + 0) \times 1 = 0$$
$$a = \beta \ : = (R(3B, \beta, T) + U(T)) \times T(3B, \beta, T)$$
$$= (600 + 0) \times 1 = 600$$
$$\pi(3B) := \beta$$

k = 1

Updated $U(s)$:

$U(1A) = (R(1A, \alpha, 2A) + U(2A))T(1A, \alpha, 2A) + (R(1A, \alpha, 2B) + U(2B))T(1A, \alpha, 2B) = (0 + 50) \times 0.4 + (0 + 400) \times 0.6 = 260$

$U(1B) = (R(1B, \beta, T) + U(T))T(1B, \beta, T) = (200 + 0) \times 1 = 200$

$U(2A) = (R(2A, \alpha, 3A) + U(3A))T(2A, \alpha, 3A) + (R(2A, \alpha, 3B) + U(3B))T(2A, \alpha, 3B) = (0 + 0) \times 0.4 + (0 + 600) \times 0.6 = 360$

$U(2B) = (R(2B, \beta, T) + U(T))T(2B, \beta, T) = (400 + 0) \times 1 = 400$

$U(3A) = (R(3A, \beta, T) + U(T))T(3A, \beta, T) = (0 + 0) \times 1 = 0$

$U(3B) = (R(3B, \beta, T) + U(T))T(3B, \beta, T) = (600 + 0) \times 1 = 600$

$U(T) = (R(T, \beta, T) + U(T))T(T, \beta, T) = (0 + 0) \times 1 = 0$

Update policy $\pi(s)$ for all states s:

$$\pi(1A) = argmax_a \sum_{s'} (R(1A, a, s') + U(s')) \times T(1A, a, s')$$
$$a = \alpha \ : = (R(1A, \alpha, 2A) + U(2A)) \times T(1A, \alpha, 2A) + (R(1A, \alpha, 2B) + U(2B)) \times T(1A, \alpha, 2B)$$
$$= (0 + 360) \times 0.4 + (0 + 400) \times 0.6 = 384$$

$a = \beta$: $= (R(1A, \beta, T) + U(T)) \times T(1A, \beta, T)$
$= (100 + 0) \times 1 = 100$
$\pi(1A) := \alpha$

$\pi(1B) = argmax_a \sum_{s'}(R(1B, a, s') + U(s')) \times T(1B, a, s')$
$a = \alpha$: $= (R(1B, \alpha, 2A) + U(2A)) \times T(1B, \alpha, 2A) + (R(1B, \alpha, 2B) + U(2B)) \times T(1B, \alpha, 2B)$
$= (0 + 360) \times 0.6 + (0 + 400) \times 0.4 = 376$
$a = \beta$: $= (R(1B, \beta, T) + U(T)) \times T(1B, \beta, T)$
$= (100 + 0) \times 1 = 200$
$\pi(1B) := \alpha$

$\pi(2A) = argmax_a \sum_{s'}(R(2A, a, s') + U(s')) \times T(2A, a, s')$
$a = \alpha$: $= (R(2A, \alpha, 3A) + U(3A)) \times T(2A, \alpha, 3A) + (R(2A, \alpha, 3B) + U(3B)) \times T(2A, \alpha, 3B)$
$= (0 + 0) \times 0.4 + (0 + 600) \times 0.6 = 360$
$a = \beta$: $= (R(2A, \beta, T) + U(T)) \times T(2A, \beta, T)$
$= (50 + 0) \times 1 = 50$
$\pi(2A) := \alpha$

$\pi(2B) = argmax_a \sum_{s'}(R(2B, a, s') + U(s')) \times T(2B, a, s')$
$a = \alpha$: $= (R(2B, \alpha, 3A) + U(3A)) \times T(2B, \alpha, 3A) + (R(2B, \alpha, 3B) + U(3B)) \times T(2B, \alpha, 3B)$
$= (0 + 0) \times 0.6 + (0 + 600) \times 0.4 = 240$
$a = \beta$: $= (R(2B, \beta, T) + U(T)) \times T(2B, \beta, T)$
$= (400 + 0) \times 1 = 400$
$\pi(2B) := \beta$

$\pi(3A) = argmax_a \sum_{s'}(R(3A, a, s') + U(s')) \times T(3A, a, s')$
$a = \alpha$: $= (R(3A, \alpha, T) + U(T)) \times T(3A, \alpha, T)$
$= (0 + 0) \times 1 = 0$
$a = \beta$: $= (R(3A, \beta, T) + U(T)) \times T(3A, \beta, T)$
$= (0 + 0) \times 1 = 0$
$\pi(3A) := \beta$

$\pi(3B) = argmax_a \sum_{s'}(R(3B, a, s') + U(s')) \times T(3B, a, s')$
$a = \alpha$: $= (R(3B, \alpha, T) + U(T)) \times T(3B, \alpha, T)$
$= (0 + 0) \times 1 = 0$
$a = \beta$: $= (R(3B, \beta, T) + U(T)) \times T(3B, \beta, T)$
$= (600 + 0) \times 1 = 600$
$\pi(3B) := \beta$

k=2

Updated $U(s)$:
$U(1A) = (R(1A, \alpha, 2A) + U(2A))T(1A, \alpha, 2A) + (R(1A, \alpha, 2B) + U(2B))T(1A, \alpha, 2B) = (0 + 360) \times 0.4 + (0 + 400) \times 0.6 = 384$
$U(1B) = (R(1B, \alpha, 2A) + U(2A))T(1B, \alpha, 2A) + (R(1B, \alpha, 2B) + U(2B))T(1B, \alpha, 2B) = (0 + 360) \times 0.6 + (0 + 400) \times 0.4 = 376$
$U(2A) = (R(2A, \alpha, 3A) + U(3A))T(2A, \alpha, 3A) + (R(2A, \alpha, 3B) + U(3B))T(2A, \alpha, 3B) = (0 + 0) \times 0.4 + (0 + 600) \times 0.6 = 360$
$U(2B) = (R(2B, \beta, T) + U(T))T(2B, \beta, T) = (400 + 0) \times 1 = 400$
$U(3A) = (R(3A, \beta, T) + U(T))T(3A, \beta, T) = (0 + 0) \times 1 = 0$
$U(3B) = (R(3B, \beta, T) + U(T))T(3B, \beta, T) = (600 + 0) \times 1 = 600$
$U(T) = (R(T, \beta, T) + U(T))T(T, \beta, T) = (0 + 0) \times 1 = 0$

Update policy $\pi(s)$ for all states s:

$$\pi(1A) = argmax_a \sum_{s'}(R(1A,a,s') + U(s')) \times T(1A,a,s')$$
$$a = \alpha : = (R(1A,\alpha,2A) + U(2A)) \times T(1A,\alpha,2A) + (R(1A,\alpha,2B) + U(2B)) \times T(1A,\alpha,2B)$$
$$= (0+360) \times 0.4 + (0+400) \times 0.6 = 384$$
$$a = \beta : = (R(1A,\beta,T) + U(T)) \times T(1A,\beta,T)$$
$$= (100+0) \times 1 = 100$$
$$\pi(1A) := \alpha$$

$$\pi(1B) = argmax_a \sum_{s'}(R(1B,a,s') + U(s')) \times T(1B,a,s')$$
$$a = \alpha : = (R(1B,\alpha,2A) + U(2A)) \times T(1B,\alpha,2A) + (R(1B,\alpha,2B) + U(2B)) \times T(1B,\alpha,2B)$$
$$= (0+360) \times 0.6 + (0+400) \times 0.4 = 376$$
$$a = \beta : = (R(1B,\beta,T) + U(T)) \times T(1B,\beta,T)$$
$$= (100+0) \times 1 = 200$$
$$\pi(1B) := \alpha$$

$$\pi(2A) = argmax_a \sum_{s'}(R(2A,a,s') + U(s')) \times T(2A,a,s')$$
$$a = \alpha : = (R(2A,\alpha,3A) + U(3A)) \times T(2A,\alpha,3A) + (R(2A,\alpha,3B) + U(3B)) \times T(2A,\alpha,3B)$$
$$= (0+0) \times 0.4 + (0+600) \times 0.6 = 360$$
$$a = \beta : = (R(2A,\beta,T) + U(T)) \times T(2A,\beta,T)$$
$$= (50+0) \times 1 = 50$$
$$\pi(2A) := \alpha$$

$$\pi(2B) = argmax_a \sum_{s'}(R(2B,a,s') + U(s')) \times T(2B,a,s')$$
$$a = \alpha : = (R(2B,\alpha,3A) + U(3A)) \times T(2B,\alpha,3A) + (R(2B,\alpha,3B) + U(3B)) \times T(2B,\alpha,3B)$$
$$= (0+0) \times 0.6 + (0+600) \times 0.4 = 240$$
$$a = \beta : = (R(2B,\beta,T) + U(T)) \times T(2B,\beta,T)$$
$$= (400+0) \times 1 = 400$$
$$\pi(2B) := \beta$$

$$\pi(3A) = argmax_a \sum_{s'}(R(3A,a,s') + U(s')) \times T(3A,a,s')$$
$$a = \alpha : = (R(3A,\alpha,T) + U(T)) \times T(3A,\alpha,T)$$
$$= (0+0) \times 1 = 0$$
$$a = \beta : = (R(3A,\beta,T) + U(T)) \times T(3A,\beta,T)$$
$$= (0+0) \times 1 = 0$$
$$\pi(3A) := \beta$$

$$\pi(3B) = argmax_a \sum_{s'}(R(3B,a,s') + U(s')) \times T(3B,a,s')$$
$$a = \alpha : = (R(3B,\alpha,T) + U(T)) \times T(3B,\alpha,T)$$
$$= (0+0) \times 1 = 0$$
$$a = \beta : = (R(3B,\beta,T) + U(T)) \times T(3B,\beta,T)$$
$$= (600+0) \times 1 = 600$$
$$\pi(3B) := \beta$$

At $k = 2$ the updated policy is unchanged. So the finaly policy is:

s	$\pi(s)$
1A	α
1B	α
2A	α
2B	β
3A	β
3B	β